1. (a) Define the least upper bound property and the completeness property of $\mathbb{R}$.
   (b) Let $x \in \mathbb{R}$. Find $\sup(A)$, where $A = \{\frac{x}{n} : n \in \mathbb{N}\}$.

2. Let $p : \mathbb{R} \to \mathbb{R}$ such that $p(x) = 1 + 2x + 3x^2$ and $\{x_n\}_{n=1}^{\infty}$ be a sequence of real numbers.
   (a) Suppose $\liminf_{n\to\infty} x_n = x \in \mathbb{R}$ does it imply $\liminf_{n\to\infty} p(x_n) = p(x)$?
   (b) Suppose $x_n \to x$, with $x \in \mathbb{R}$. From the definition of limit show that $p(x_n) \to p(x)$.

3. Please decide whether the following statements are true or false. If you decide that a statement is true then please provide a proof. If you decide that a statement is false then please provide a counter-example with justification.
   (a) If $I_n, n \geq 1$ is a sequence of nested intervals (i.e $I_{n+1} \subset I_n$ for all $n \geq 1$) then $\cap_{n=1}^{\infty} I_n \neq \emptyset$.
   (b) Suppose $\{x_n\}_{n=1}^{\infty}$and $\{y_n\}_{n=1}^{\infty}$are such that for every $\epsilon > 0$ there is an M such that $\mid x_n - y_n \mid < \epsilon$ for all $n \geq M$. $\limsup_{n\to\infty} x_n = \limsup_{n\to\infty} y_n$.
   (c) There is no ordering on the field of complex numbers such that it becomes an ordered field.
   (d) Let $\{x_n\}_{n=1}^{\infty}$and $\{y_n\}_{n=1}^{\infty}$be two convergent sequences of real numbers such that $x_n < y_n$ for all $n \in N$. Then $\lim_{n\to\infty} x_n < \lim_{n\to\infty} y_n$.

4. Find the radius of convergence of the power series $\sum_{n=1}^{\infty} \frac{n^n}{n!} x^n$.

5. Let $a, b, c \in \mathbb{R}$. Suppose $\{x_n\}_{n=1}^{\infty}$ is a sequence of real numbers such that

$$x_n = \frac{a}{n} + \frac{b}{n+1} + \frac{c}{n+2}, \quad \forall n \in \mathbb{N}$$

   Decide whether the series $\sum_{n=1}^{\infty} x_n$ converges or not.